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Deterministic Chaos and Natural Phenomena

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The natural time series f_0F_2, F10.7, and AE are analyzed and low-dimensional attractors are found, characterized by the correlation dimension and the lower bound of the Kolmogorov entropy. Sources of noise in natural time series are discussed and the concept of extended systems is introduced and used to explain why the number of data required to calculate the correlation dimension of natural time series is higher than that reported by other authors.

KEY WORDS: Chaos; extended systems; noise; time series.

1. INTRODUCTION

The vast majority of published literature on chaos is concerned with numerical or laboratory experiments, in spite of the professed belief that natural phenomena are nonlinear. Among the few exceptions are papers by Nicolis and Nicolis,[(1)] Kurths,[(2)] and Romanelli *et al.*[(3)]

In this paper we present some results on natural time series and explain some of the difficulties associated with them.

2. DATA PRESENTATION

We shall discuss three different types of natural time series. They are:

f_0F_2: Hourly values of the ionospheric critical frequency. This value is obtained from continuous measurements and pertains to a *particular* time and place.

F10.7: Daily values of solar radio flux at 10.7 cm. This value is representative of the daily solar activity. It is thus an *integration* of phenomena in the visible side of the sun as received on earth during a day.

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AE: Auroral electrojet activity is defined by Davis and Sugiura[4] as an *index* obtained from geomagnetic data of selected sites of the northern hemisphere that is considered to give a reliable auroral activity information on global scale. In our case, it is an hourly average related to the deviation of the horizontal component of the geomagnetic field.

Some difficulties associated with these time series are:

(a) Sampling period: generally lower than one sample per hour. As a consequence, relatively short series may take a long time to obtain, giving rise to questions of whether the (generally unknown) parameters that control the phenomena may have changed during the period of observation.

(b) Noise: These data include noise arising from different sources.

(i) Measurement errors: typically of the order of 1 %. This is equivalent to roundoff error in the data obtained from numerical experiments.

(ii) Transmission error: that added to a measured signal between the phenomena and the observer. For example, in the case of F10.7, the phenomena occur in the sun and are measured on earth. This error is equivalent to adding random noise to numerical experiments data.

(iii) *Intrinsic* noise: when dealing with extended systems such as those dealt with in this paper, they should be considered as a collection of similar dynamical subsystems, each one contributing with "noise" (due to the phenomena themselves) and their random superposition.

(c) Nonrepeatability of the observations.

Other related problems are: missing observations, nonavailability, etc.

In order to determinate if the variability observed in the data corresponds to deterministic behavior in its origin or may be attributed to random noise, we have analyzed the natural time series mentioned above.

If deterministic chaos is observed, this result is useful because it shows that nonlinear equations are necessary to model the system and puts an upper bound on the number of them.

3. METHOD OF ANALYSIS

When analyzing experimental time series most of the N variables of the system under study are usually unknown or unavailable. Therefore, the question is whether and how it is possible to substitute the missing information.

Takens[5] has found that instead of $X(t)$ and its derivatives, it is easier to work with $X(t)$ and the set of variables obtained by shifting the original series by fixed lags or delay time τ. This provides enough information to reconstruct, from a one-dimensional space, a multidimensional phase space of the dynamical system.

It is interesting to reconstruct the phase space because the nature of the attractors provides information on the time behavior of the variables and on the nature of their coupling.

It is necessary now to characterize the complexity of the dynamics more precisely and, in particular, discriminate whether the observed fluctuations in the data are due to random noise or belong to some deterministic behavior.

The structure of the attractor is inferred from the correlation dimension and the entropy K_2.[6] The so-called integral correlation function $C_d(r)$ is given by

$$C_d(r) = (1/N^2) \sum_{i \neq j} \theta(r - |x_i(t) - x_j(t)|)$$

for a d-dimensional space, where θ is the Heaviside function, N is the total number of data, and X_i stands for a point of phase space. The integral $C_d(r)$, for small r, scales as $C_d(r) \sim r^{\upsilon}$. From the slopes of the log–log plots of $C_d(r)$ versus r for different values of d, values of υ as a function of d can be derived. The saturation value of the υ versus d plot is the correlation dimension D_c.

If X_i is random noise, the correlation integral scales as $C_d(r) \sim r^d$ and there is no saturation.

A lower bound of the Kolmogorov entropy K_2 is found from

$$K_2(r) = (1/\tau) \log[C_d(r)/C_{d+1}(r)]$$

$K_2 > 0$ for deterministic chaos. If the system evolves periodically, $K_2 = 0$ and for stochastic systems, $K_2 = \infty$.

It is an open question what is the minimum amount of data required for the Grassberger–Procaccia method to work.

Several authors[6,7] say that N should be very large. But Abraham *et al.*[8] have done numerical experiments and found that 600 data points are enough to obtain reasonable results. The results of a previous work by Romanelli *et al.*[3] indicate that when dealing with natural time series about 1700 data points are required. We believe that the difference in the number of necessary points can be found in the fact that our natural series originated in extended sources, as was discussed in Section 2.

4. RESULTS

The series used in this work are:

(a) Hourly values of the ionospheric critical frequency f_0F_2 for Argentine Island (65.25°S, 64.27°N) for the years 1977–1978.

Table I. Characteristics of the Time Series Analyzed and the Values of the Dimensions Found

Parameter	Sampling	Data points	D_c	K_2
F10.7	Daily	1,826	3.5	0.07
AE	Hourly	2,928	3.3	0.08
f_0F_2	Hourly	17,280	3.4	0.04

(b) Daily values of solar flux at 10.7 cm from January 1973 to December 1977.

(c) Hourly values of the auroral geomagnetic index AE from September to December 1983.

The values obtained for the correlation dimension D_c are given in Table I. It can be seen in that all cases we are in the presence of chaotic attractors.

It may be surprising at first glance that similar results are obtained from different physical phenomena, but this is due to the fact that these systems are closely related to the Navier–Stokes equations.

Therefore we conclude that in the three types of natural time series we have analyzed, low-dimensional chaotic attractors are present, and in all cases four equations are needed in order to model the system.

Natural phenomena have to be considered as a collection of dynamical chaotic subsystems. The theoretical treatment of such systems is the object of future work.

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